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A single-risk statistical model for evaluating insurance demand

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Abstract :

This paper introduces a single-risk model for statistically analyzing the problem of evaluating the "personal risk" in a self-insurance situation. The statistical formulation of this model is based upon "a probabilistic quantification" of the risk in such type of insurance. A set of reasonable assumptions is assumed, by which the problem has been casted in a simple "probabilistic structure". Analytical solution for such probabilistic formulation has been developed for the determination of the "optimal demand" for insurance. To realize this, the above-mention probabilistic structure is viewed as a traditional maximization setting, in which a particular form for the utility function has been implemented. Also, the analytical solution confirms the dependence of the demand for insurance on the value of the initial wealth.

Introduction :

This paper provides an answer to the problem of assessing the "risk" of self-insurance. Having proposed a set of assumptions, a statistical model for single-risk insurance is constructed, by which a "probabilistic answer" is provided for this problem. In fact, it is difficult to "quantify" the risk of such insurnce because "personal" decisions under conditions of risk and incertitutde depend upon the influence of many conflicting "factors". This "uncertain" behavioral attitude towards "risk", poses a problem in quantifying it. This, in fact, represents a "Rationale" for using some – simple – analytical technique for "quantifying" the uncertainty of some of the decisional factors that are affecting the determination of the self-insurance "demand".

In this context, one must be interested in the "relevance" of these decision factors in uncertain situations. This can be realized by analyzing a problem of "assurance against regret", using a simple proposed model for determining the insurance demand.

This paper is organized as follows. Section 2, introduces some assumptions, upon which the model is constructed. These assumptions have been classified into two types, mainly, behavioral and distributional. Section 3, reports the proposed criteria that are employed to derive the optimal demand for self-insurance setting. Section 4, discusses an evaluation of the derived model. This includes the main contributions of the model as well as its basic drawbacks. Section 5, introduces a numerical example which gives a practical meaning of the main findings of the model. Section 6, proposes some constructive suggestions, by which a more "general solution" could be achieved for solving the multiple-risk insurance problem.

2- Model description :

This section introduces some realistic "assumptions", by which the proposed model will be constructed. The basic criteria for evaluating the expected utility wealth function E(W) are mentioned. Some useful relations between the demand for insurance (D) and the value subject to destruction (V) are discussed in cases of total and partial destruction hypotheses.

(2.1) : Model Behavioral Assumptions :

A 1 : The simple case of an individual is considered, in which he possesses a certain quantity of financial wealth, denoted by (W_0).

A 2 : He possesses a real estate, of a known value, denoted with (V), that is subject to the "risk" of destruction.

A 3 : The "risk" of destruction is the only source for incertitude.

A 4 : The person decides to buy an insurance policy to get protection against the "destruction risk". The person will pay the value (M) of the policy (insurance premium) "in return for" getting a compensation of a value (D)

(2.2) : Intuitive Relational Assumptions :

A 1 : Assume that the probability of a disaster's occurrence is "p".

A 2 : Considering the proportions (V) and (D), there epists "only" one of the following relations.

(i) Totally-covered damage : This will occur when (D = V). thus the person is compensated with the "same" value of his "suffered loss". This means that his financial damage is "zero", but he might be suffering a "moral loss".

(ii) Partially-covered damage : This situation corresponds to a partial – insurance case, when (D > 0) and (D < V). thus, the insurer is covering only a port (D is a part of V) of the insured person's loss who receives a positive (D > 0) compensation.

(iii) The value of (D) equals zero, if the person is "not" insuring himself against the "risk", and he will be bearing the loss entirely.

(iv) A theoretical case may be viewed, if we have the postulated case that (D > V). This means that the insured person receives a "higher" compensation than his suffered damage. In this case he might be tempted to provoke the disaster, so the insurance companies are totally "excluding" this possibility from their proposal of contracts.

A 3 : Assuming that the insurance premium is denoted by (M). this depends upon the following three elements :

(i) The insurance amount (denoted by D) or the value of the claimed compersation;

(ii) The probability of risk-occurrence (p); and

(iii) The costs of the insurer. Knowing that the insurance premiums (M) are representing the insurer's revenues, thus it must have – in advance – the recovery of the expenses and also an "eventual profit". This leads to the fourth assumption.

A 4 : Assume that, for each monetary unit of compensation, the "expenses" of the insurer are called the "charging rate" of the insurance premium, and denoted by (λ).

3- Issues on Model Building :

This section starts with reviewing the criteria, that are used for building the model. Some distributional relations are developed, depending upon the shape and the behavior of the utility function T(w). This leads us to construct an "upper bound" for the charging rate (λ). In this context three cases are given for different values of (λ).

(3.1) : General Framework :

Here the major criteria, which have been implemented in the model, will be briefly reviewed. Firstly, an evaluation of the expected utility of wealth E(w) is performed, and the "optimal selection" is found to be (D) less than (V), which refers to "a partial insurance".

In this context, some mathematical assumptions, are made concerning the "behavior" of the utility function T(w). They are

A1 : The utility function T(w) is "twice" differentiable.

A2 : Assume that : $T^{\backslash}(w) > 0$, which means that "more" wealth is preferred to "less" (non-satiation).

A3 : Assume that $T^{\backslash\backslash}(w) < 0$, which means that the individual has diminishing marginal utility.

The above set of assumptions, means that the individual is "risk averse". Thus the "optimal selection is (D = V) which indicates a "total insurance" decision.

For modeling purposes, a charging rate (λ) is defined, by which a quantification of the "Partial risk" is derived. Secondly, a decision for the "optimal" choice of the insurance demand (D^*) is made via the "profitability-risk" criterion (or the mean – variance criterion). A coefficient of the owner's attitude towards risk (r) is considered to quantify the risk in this case.

(3.2) : Basic foundations of the model :

From the assumptions mentioned above, one may represent the insurance premium as:

$$M = (1 + \lambda)\, p\, D \qquad (3.1)$$

If the value of (λ) equals zero, which means that the insurance company would have "no" costs, then the insurance premium would be equal to the "actuarial" value of the compensation. This is because the compensation D is an aleatory variable (D, 0) with probability distribution [D, 0; p , 1- p], and whose mathematical expectation E(D) = p D.

Now, after the occurrence of the disaster, the land lord will be interested in his "final wealth", denoted by (W). This is also, an aleatory variable (W_1, W_2) with probability (p, 1 – p), where :

$$W_1 = W_0 + D - M$$
$$W_2 = W_0 + V - M \qquad (3.2)$$

Adopting Bernoulli's Model, the owner is trying to select the level of compensation (D^*) which maximizes the utility expectation of his final wealth E {T(w)}. This means that, the model of his "demand" for insurance postulates that the maximum of E{T(w)}, where T(w) is the utility function of wealth, being given as an "increasing" function of the wealth's level and "concave" in the variable (D). Having known that $D \leq V$, it results that

$W_1 \leq W_2$, and

$$E\{T(w)\} = p.\, T(w_1) + (1 - p)\, T(w_2) \qquad (3.3)$$

The concellation of the first degree derivative, leads to the "improvement condition" :

$$p\,[1 - (1 + \lambda)\, p]\, T^{\backslash}(w_1) + (1 - p)\,[-(1 + \lambda)\, p]\, T^{\backslash}(w_2) = 0 \qquad (3.4)$$

Taking into consideration the hypothesis made on the utility function T(w) and that the second factor is negative the above condition (3.4) can be satisfied only if :

$[1 - (1 + \lambda)\, p] > 0$, which means that

$$\lambda < (1 - p) / p \qquad (3.5)$$

The relation (3.5) is a derived "upper bound" for the charging rate (λ), which appears to be "depending" upon the disaster's probability of occurrence (p).

(3.3) : Determination of the insurance decision :

Here we discuss the role of the charging rate (λ) in determining the insurance decision. This role extends to include three possible insurance decision, which are given by further investigation of the relation (3.5).

(i) If we suppose that;

$$\lambda > (1 - p) / p, \qquad (3.6)$$

which is indicating a "high" charging rate (λ), that determines a "zero" demand for insurance.

(ii) If the charging rate (λ) satisfies the inequality

$$0 < \lambda < (1 - p) / p,$$

which is representing "a responsible" charging rate and will be leading a rational owner to a "partial insurance decision".

(iii) The null charging rate ($\lambda = 0$) the insurance premium $M = (1 + \lambda)\, p\, D$, becomes $M = pD$, and the improvement condition (3.4) becomes :

$$p\,(1 - p)\,[T^{\backslash}(w_1) - T^{\backslash}(w_2)], \qquad (3.7)$$

If the person is a "risk-aversion" one, and then by recalling the mathematical assumptions in (3.1), then the following expression is positive, i.e.

$$[T^{\backslash}(w_1) - T^{\backslash}(w_2)] > 0\, . \qquad (3.8)$$

This means that the derivative of E{T(w)} with respect to (D) is positive. If ($W_1 = W_2$), the expression in (3.8) cancels itself, meaning that (D) equal (V).

The final conclusion is that when ($\lambda = 0$), the "optimal" decision is the "total insurance" (D = V).

4- Derivation of the model :

In this section, we implement the previously mentioned aspects to derive the expectation, and the variance of the individual's wealth. Then, by assuming a logarithmic utility function a "precise" value (D^*) of the demand for insurance will be established (or determined).

(4.1) Measuring the risk-attitude :

Within the previous framework, given in (3.1), some descriptive criteria have been used to assess the result of a certain "insurance – decision" as well as the "individual's attitude" towards risk.

Firstly : If the initial wealth is (W_0), and if the individual is considering "the profitability – risk" criterion in taking his decision, then:

$$E\{W\} = pW_1 + (1 - p)\, W_2 = W_0 + (1 - p)\, V - \lambda\, p\, D \qquad (4.2)$$

Also, the wealth variation (dispersion) is

$$\begin{aligned} Var(W) &= p[W_1 - E(W)]^2 + (1 - p)\,[W_2 - E(W)]^2 \\ &= p\,(1 - p)\,(D - V)^2 \qquad (4.3) \end{aligned}$$

From equation (4.3) we notice that if (D) increases the owner's "risk" will be decreased. Thus, the risk measured by (4.3) – reaches its "maximum" when (D) equals zero (non-insurance case). Also, Var(W) "decreases" until it reaches zero when (D) equals (V) which means a "total insurance" case.

Secondly : the appreciation function f(E, Var) of the "risky" situation, can be used to "quantify" such risk by implementing the coefficient (r) of the owner's attitude towards risk. This is done as follows :

$$f(E, Var) = E - r(Var)$$

$$= [W_0 + (1 - p)\, V - \lambda\, p\, D] - r\,[p(1 - p)\,(D - V)^2] \qquad (4.4)$$

Now, deriving (f) against (D) and cancelling the derivative, the following equation is obtained,

$$-\lambda\, p - 2\, r\, p\, (1 - p)\,(D - V) = 0 \qquad (4.5)$$

with the solution :

$$D^* = V - \lambda / 2r\,(1 - p) \qquad (4.6)$$

Using (4.6) we can describe the owner's attitude towards risk as follows :

(1) If ($r = 0$) this means that the owner is neutral to risk. This means that the utility function T(w) is a linear function whose derivative is a constant value.

(2) If ($r > 0$), then the owner is "risk – aversion". This case corresponds to the "null" charging rate (λ) in using the utility function T(w) in the analysis of the owner's insurance decision.

(4.2) : Exact Assessment of the demand for insurance :

A major drawback in the above obtained expressions (4.2), (4.3) and (4.4) is that they don't show the "influence" of the initial wealth (W_0) on "the optimal amount" of compensation (D). Thus, in this section we implement the "criterion" of the expectation of utility E{T(w)} as a way of demonstrating the dependence of the demand for insurance (D) on the initial wealth (W_0).

The Determination of the "precise" sum for the compensation - demand for insurance (D) – requires a "known" functional form "for the utility function T(w) of the owner's wealth.

Knowing that the expectation is a "linear operator", this gives a "rationale" for choosing the "logarithmic" form "for the utility function. This in because any affine "transformation of T", will preserve the same features of "T".

Under the assumption that

$$T(w) = \ln(w) \qquad (4.7)$$

And by making some mathematical manipulations the proposed model gives us the following "precise" or "compact" form of the demand for insurance :

$$D = \frac{-\lambda W_0 + [1-(1+\lambda)p]\,V}{(1+\lambda)[1-(1+\lambda)p]} \qquad (4.8)$$

Relation (4.8) confirms the "dependence" of the demand for insurance (D) on the initial wealth (W_0), as well as, the "inverse relation" between them. This is because if ($\lambda > 0$) then (D) is a "decreasing" function of W_0. The above relation is true if all the factors in (4.8) are kept unchanged.

Also, equation (4.8) is analytically beneficial in explaining the relation between (D) and (V). Thus, (4.8) assures the following properties:

- if $\lambda = 0$, then $D = V$;
- if $\lambda > 0$, then $D < V$
- Knowing that the derivative of (D) with respect to (V) is positive, ie.: $\frac{dD}{dV} = 1/(1-\lambda) > 0$

5- Numerical example :

It is assumed that there exists a landlord, who disposes a stable financial wealth of W_0 = 5000 USD and an immovable good of the value V = 15000 USD, is undergoing a fire risk. The probability that the disaster is total is estimated at p = 1/1000. To protect himself against the danger, the landlord stopped at an insurance company that utilizes a charging rate of λ = 0.2. The landlord is risk aversion and he evaluates his economic state with a logarithmic utility function. Then using equation (4.2) we get

$$E\{W\} = W_0 + (1 - p)\,V - \lambda pD =$$
$$= 5000 + (1 - 1/1000)\,15000 - 0.2(1/1000)D$$
$$= 19985 - 0.0002D \qquad (5.1)$$

Also, by recalling equation (4.3) we get

$$Var(W) = p(1 - p)\,(D - V)^2 =$$
$$= 0.000999\,(D - 15000)^2 \qquad (5.2)$$

and the optimal value of the demand for insurance by using (4.6) is

$$D^* = V - \lambda/2r\,(1 - p) = 15000 - 100/999\,r \qquad (5.3)$$

By using (4.8) we can obtain the demand insurance under the logarithmic utility function as D = 11665,665 USD.

Buying the insurance policy for which he pays the premium of

$M = (1 + \lambda)\,pD = 1.2\,(1/1000)\,11665{,}665 = 14$ USD on each year. In this case the owner assures himself that, if the disaster will happen, the insurance company will compensate him with approximately 78% of the total value of his loss.

6- Suggestions and concluding remarks :

Maximization of the statistical expectation of utility E{T(w)} is the fundamental basis for building the proposed model to represent how an individual make a decision under the condition of risk (or under the individual's particular "probability beliefs". This model is a single-risk model which assumes that the risk of destruction is the only source for incertitude. One can suggest that modeling a multiple risk situation would be more realistic and will give more general results.

Another suggested viewpoint can be introduced concerning the wealth (W) which has been dealt with as "monetary wealth". It will be more realistic to consider the "real" wealth in proposing the modeling assumptions.

Thirdly, concerning the expected utility approach, there exists a shortcoming which needs further investigation. That is, do individual really behave as if they maximize E(T), even in a single period context ?.

Fourthly, it was supposed implicitly that the landlord and the insurance company consider the destruction risk (measured by λ) in the same manner and give to the probability p the same value. Unfortunately, this hypothesis is not always verified, because of the asymmetry of information between the two. The practical situation (in the European insurance market) insurance companies are calculating the insurance premium (M) as a percentage of the insured amount (D) or in other way the insurance contract considers the insurance premium $M = \alpha D$. Knowing that (α) must be equal to $(1 + \lambda)p$, the insured may have to perform some different simulations in order to assess the real values of the charging rate λ and of the probability p.

As a final concluding remark, "an analogy" between the charging rate (λ) and coefficient of the owner's attitude towards risk (r) can be realized from the proposed model. This will be confirming a "similar effect" of these two factors on the determination of the optimal demand for insurance (D^*).

References :

1- Artzner, P. and Delbaen, F. (1995) Default risk insurance and incomplete markets, Mathematical Finance, 5, 187-195.

2- Booth P.M. (1997) The Analysis Of Actuarial Investment Risk, Actualial Research Paper No. 93. City University, London U.K.

3- Hand, D.J. and Jacka, S.D. (2002) – Statistics In Finance. London. U.K : Wiley.

4- Herstein, I.N. and Milnor, J. (1953) An axiomatic approach to expected utility. Econometrica, 21, 291-297.

5- Hodges, S.D. and Neuberger, A (1989) Optimal replication of contingent claims under transactions costs. The Review of Futures Markets, 8, 223-239.

6- Ingersoll, J.E (1987) theory of Financial Decision Making. New Jersey : Rowman and Littlefield.

7- Knight F. (1971) risk, uncertainty and profit, University of Chicago Press.

8- Machina, M. (1982) Expected utility analysis without the independence axiom. Econometrica, 0, 227-323.

9- Nielsen, L. T., Saá-Requejo, J. and Santa Clara, P. (1993) Default risk and interest rate risk : the term structure of default spreads. Working Paper, INSEAD, 77305 Fontainebleau Cedex, France.

10- Pratt J.W. (1964). Risk Aversion in the small and in the large. Econometrica, 32 (1-2), 122-136.

11- Smith, A.D. (1996). How actuaries can use financial economics. British Actuarial Journal, 2, 1057-1193.

12- Wilkie A.D. (1995). More on a stochastic model or actuarial use. British Actuarial Journal, 1, 777-964.